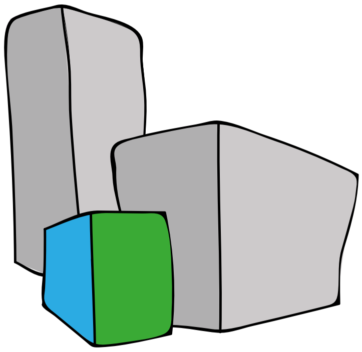
Christmas Colours

The elves in Santa's workshop are getting ready to wrap all the Christmas presents. Santa has decided he wants them to wrap the boxes a little bit differently this year.

The boxes must be wrapped so that any 2 faces of the box which have an edge in common have different coloured wrapped paper on them.



If all the presents are in cuboid boxes, what is the smallest number of wrapping paper colours the elves will need?

HINT: Think about the number of faces a cuboid is made up of and where the faces, edge and vertices meet. It might help to draw the net of a cuboid.

Extension:

In the card-making department, other elves are designing a pattern for this year's Christmas card.

To make sure the pattern is interesting, they decide that any two areas that share a boundary cannot be the same colour. (If two areas only meet at a point, they can be the same colour).

But they also don't want to have too many colours on their card, so they only add a new colour in to the pattern if they can't make the pattern without it.

What is the maximum number of colours they need, regardless of the pattern they make?

HINT: Try drawing some of your own patterns.
Can you find one that needs 2 colours, how about 3, 4 etc?

Complete Mathematics

Christmas Colours
Teacher Notes

Teacher Notes

Strand: Geometry And Measures
Group: Properties of 3D shapes
Suggested Age: 10+

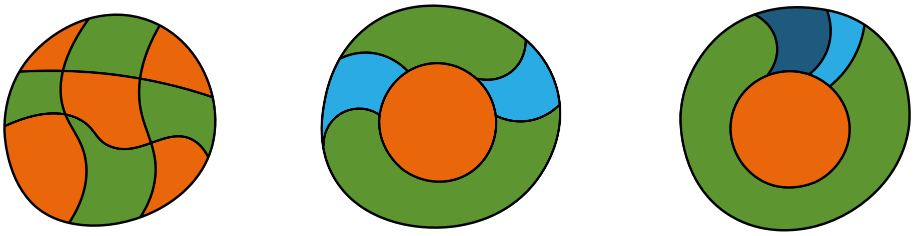
This problem is abstract, but a knowledge of nets and elevations will be an advantage. Students should also know the cuboid has 6 faces and they should know the terms faces, edges and corners (vertices). It may help them to see the problem better if they draw out the net of a cuboid (the simplest would be a cube).

The elves will need 3 colours.
Consider taking one corner of a box - three of the faces meet there. Each pair has an edge in common so we need three different colours. There are 6 faces on the cuboid and provided that the elves put the same coloured wrapping paper on the opposite faces, they will need no more colours, as opposite faces do not share an edge.

Extension

Following the rules, the elves will be able to make any pattern using a maximum of 4 colours.

From the previous activity, pupils should have a general idea of what is going on, so a good starting point is to try and create a pattern that needs a certain number of colours. This is possible for 2, 3 and 4 colours (like the examples below).



When they come to 5 colours, they will find that any pattern they create can actually be coloured with fewer colours. They may need to have a partner check their pattern to see this.
Encourage pupils to try a range of different types of patterns to convince themselves that 4 is the maximum number of colours that are needed for any pattern.

This is called the 'Four Colour Theorem' which was first stated in 1852, and only proven in 1976, using a computer. The theorem states that any map or pattern can be coloured using only 4 colours, so that any regions with a common boundary are different colours (apart from regions that only meet at a single point).

More background information about the history of the problem can be found here:
https://nrich.maths.org/6291